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PERFORMANCE OF PARALLEL LINEAR ITERATIVE PRECONDITIONERS AND SOLVERS FROM A FINITE ELEMENT MODEL OF WOODY VEGETATION ON LEVEES

Fred T. Tracy*, Maureen K. Corcoran† and Hung V. Nguyen*

* U.S. Army Engineer Research and Development Center
Information Technology Laboratory
3909 Halls Ferry Road, Vicksburg, MS 39180 USA
e-mail: Fred.T.Tracy@usace.army.mil

† U.S. Army Engineer Research and Development Center
Geotechnical and Structures Laboratory
3909 Halls Ferry Road, Vicksburg, MS 39180 USA
e-mail: Maureen.K.Corcoran@usace.army.mil

* U.S. Army Engineer Research and Development Center
Information Technology Laboratory
3909 Halls Ferry Road, Vicksburg, MS 39180 USA
e-mail: Hung.V.Nguyen@usace.army.mil

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Summary. This research provides the results of testing common Krylov subspace linear iterative solvers and preconditioners in a parallel environment using the Portable, Extensible Toolkit for Scientific Computation (PETSc) software on extremely heterogeneous, unsaturated flow problems. A three-dimensional (3-D) model of a levee with a root zone embedded at the toe served as the test problem, and the runs were done on the Cray XE6 with 128 cores.

1 INTRODUCTION

The record-breaking floods of 2011 brought the importance of the levee systems protecting cities and farmland near our rivers to the forefront. Sand boils from underseepage have become common terminology for many Americans. This research illustrates the importance of using high performance, parallel computing to analyze levee performance during flood conditions. Both two-dimensional (2-D) and three-dimensional (3-D) numerical models of levees with embedded tree root systems based on the finite element method were built and run. Specifically, an initial analysis of the effect of woody vegetation on levees with respect to the initiation of internal erosion and the variability of hydraulic conductivity was completed.

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14. ABSTRACT

This research provides the results of testing common Krylov subspace linear iterative solvers and preconditioners in a parallel environment using the Portable, Extensible Toolkit for Scientific Computation (PETSc) software on extremely heterogeneous, unsaturated flow problems. A three-dimensional (3-D) model of a levee with a root zone embedded at the toe served as the test problem, and the runs were done on the Cray XE6 with 128 cores.

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These types of simulations create significant challenges when using a time-stepping algorithm such as Euler's implicit algorithm. This is because an ill-conditioned linear system of equations must be solved at each nonlinear iteration for each time-step. Coincident to this effort, a basic research project was conducted to determine the best Krylov subspace iterative linear solvers for navigation locks and watershed simulations[1,2,3,4]. This research adds to the knowledge base by focusing on solvers for woody vegetation on levees. The Portable, Extensible Toolkit for Scientific Computation (PETSc)[5] library was used in both efforts.

2 LEVEE MODEL

Fig. 1 shows the cross section with material types for the Pocket Levee in Sacramento, CA, and Table 1 gives the saturated hydraulic conductivities. Fig. 2 provides an example of the 2-D mesh that was used, and Fig. 3 shows a 3-D model of the levee that was generated by extruding the 2-D cross section in the third dimension multiple times.

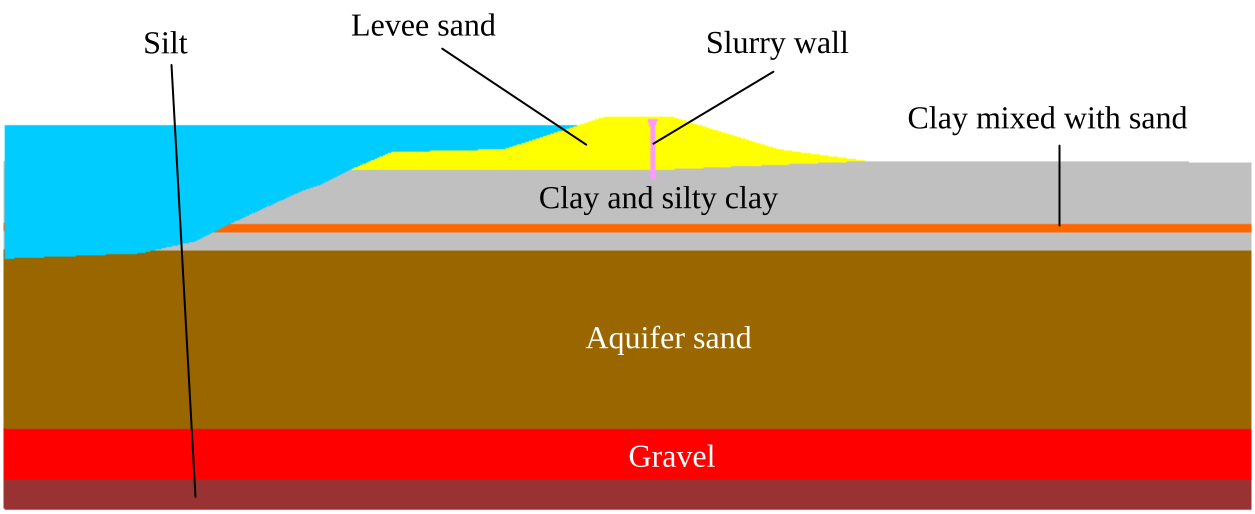

Figure 1: Cross section of the Pocket Levee, Sacramento, CA

Material	k_H (cm/sec)	k_H (ft/day)	k_V (cm/sec)	k_V (ft/day)
Levee sand	8.00×10^{-3}	22.7	2.00×10^{-3}	5.67
Clay and silty clay	8.00×10^{-4}	2.27	2.00×10^{-4}	0.568
Clay mixed with sand	3.00×10^{-5}	0.085	1.00×10^{-5}	0.0283
Aquifer sand	8.00×10^{-2}	226.7	2.00×10^{-2}	56.7
Gravel	2.00×10^{-2}	56.7	2.00×10^{-2}	56.7
Silt	1.00×10^{-4}	0.283	1.00×10^{-4}	0.283
Slurry wall	1.00×10^{-6}	0.00283	1.00×10^{-6}	0.00283

Table 1: Hydraulic conductivities used for materials identified in the cross section shown in Figure 1

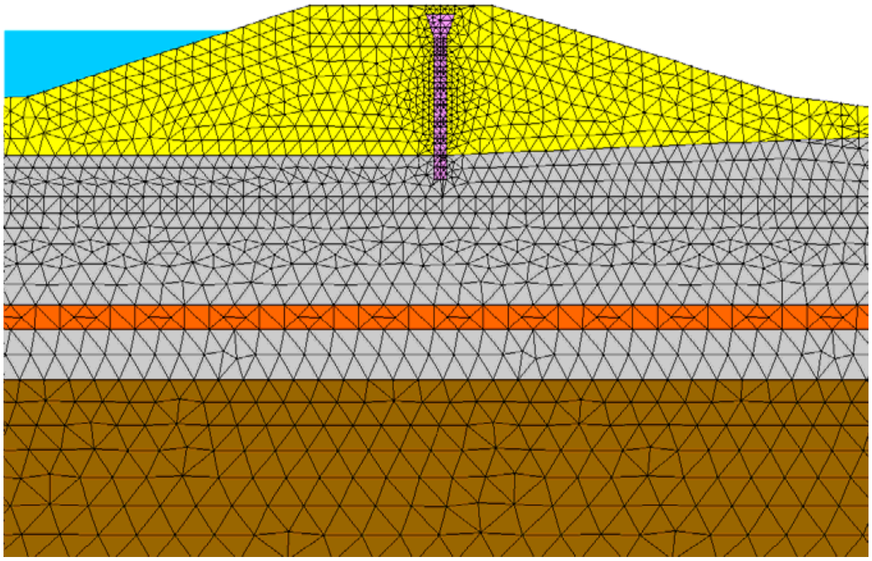

Figure 2: Portion of finite element mesh of levee cross section

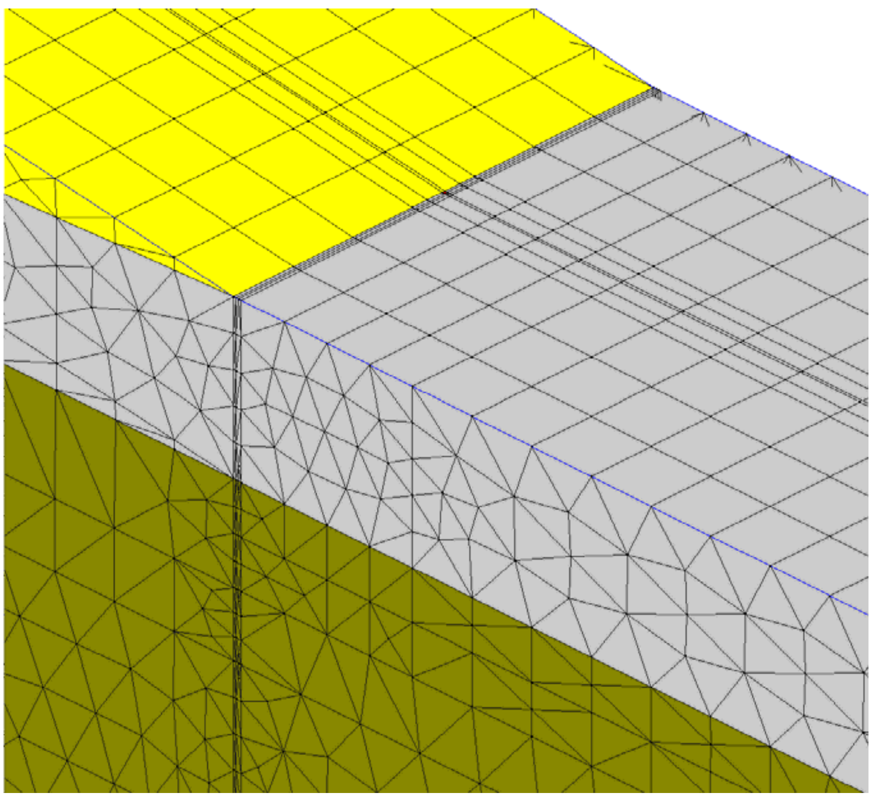

Figure 3: Example of 3-D mesh generated from 2-D model

3 ROOT ZONE MODEL TYPES

3.1 Changes in hydraulic conductivity

As shown in Fig. 4, tree root zones for the 2-D analyses were modeled by creating near rectangular 6-ft × 5-ft areas as estimated from geophysical surveys[6]. Hydraulic conductivity for a given tree root zone was assigned to each element using

$$k_{veg} = \beta k_{no-veg} \tag{1}$$

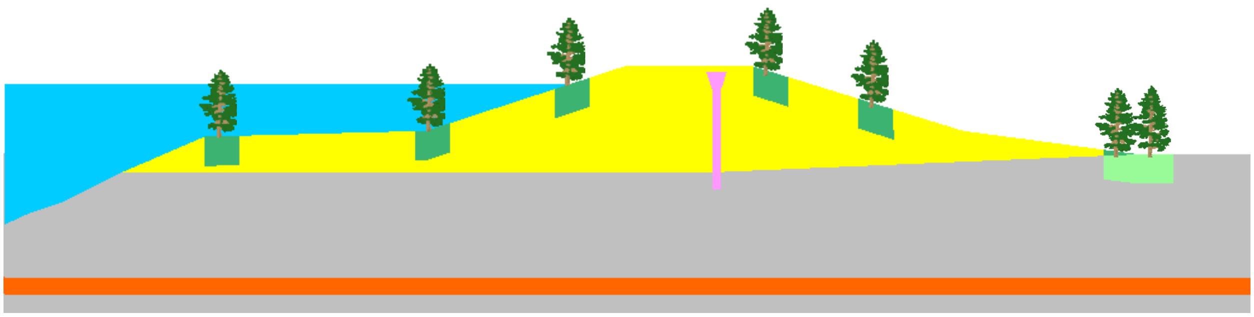

Figure 4: Tree placement for Pocket Levee, Sacramento, CA

where k_{veg} is the modified hydraulic conductivity, β is a positive parameter with recommended values[6] of $0.01 < \beta \leq 100$, and k_{no-veg} is the original hydraulic conductivity without the tree.

3.2 Macropore heterogeneity

Another way of modeling root systems is to extend the root zone concept by filling the root zone with small triangular elements in 2-D and prism elements in 3-D of approximately 1 inch in each direction, but for this model, the hydraulic conductivity of each small element is randomly varied. For the i^{th} small element in the root zone, a random number was generated, $0 \leq \zeta_i \leq 1$, then β_i was computed for the i^{th} element using

$$\beta_i = 10^{4\zeta_i - 2} \tag{2}$$

Fig. 5 shows this concept as implemented in 3-D. In fact, this dataset provides significant challenges, and it is the one used for testing to determine the best linear solver and preconditioner for seepage in levees with woody vegetation. The mesh has 3,017,367 nodes.

4 COMPUTATIONAL CHALLENGE

The computational challenges for this dataset are summarized as follows:

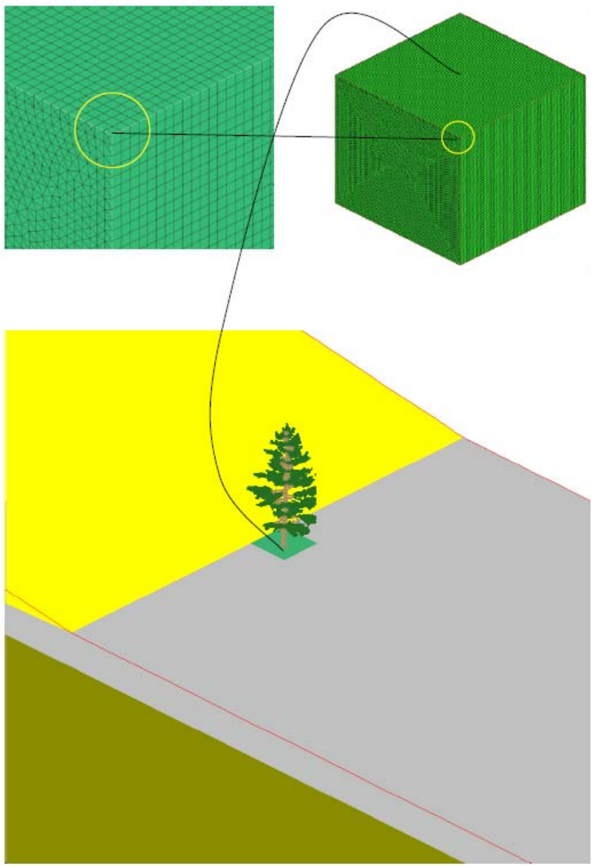

Figure 5: Small 3-D 1-inch elements in root zone

- Hydraulic conductivity in the saturated flow region (k_s) for different soils (such as sand and clay) can vary several orders of magnitude. The values shown in Table 1 vary four orders of magnitude.
- Horizontal hydraulic conductivity is often four times or more greater than vertical hydraulic conductivity.
- For flow in the unsaturated zone, hydraulic conductivity is modeled by

$$k = k_r k_s \tag{2}$$

where k_r is the relative hydraulic conductivity and $0 < k_r \leq 1$. k_r varies several orders of magnitude.
- The heterogeneous root zone shown in Fig. 5 can have adjacent elements in which the hydraulic conductivity can differ by as much as four orders of magnitude.
- All the above points combine to create ill-conditioned linear systems of equations that were solved at each nonlinear iteration.

5 RESULTS

Four linear iterative solvers and four preconditioners were run using 128 cores on the Cray XE6. The system of equations is symmetric and positive-definite. The solvers used are conjugate gradient (CG), conjugate residual (CR), bi-CG stabilized (BI), and Generalized Minimal Residual (GM). The preconditioners used are Jacobi, block Jacobi, Additive Swartz Method (ASM), and Boomer Algebraic Multigrid (AMG). Fig. 6 shows a plot of the timing results, and Table 2 gives details on iteration counts and running times. Runs were made without (labeled 0) and with (labeled 1) the presence of a tree. A dash indicates that the solution failed either by not converging after 100,000 iterations or the solver gave error messages.

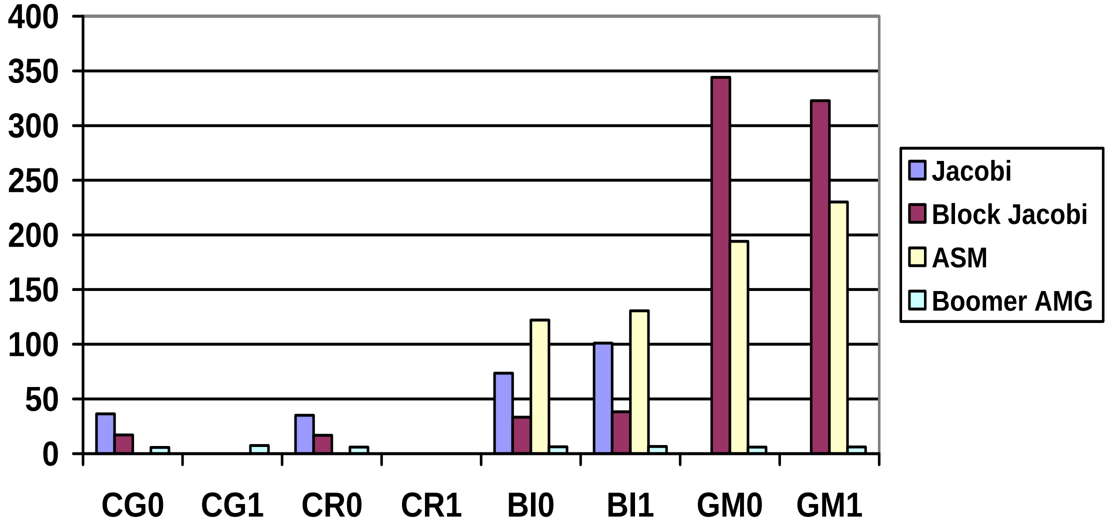

Figure 6: Running times (sec) for preconditioner and solver options

6 CONCLUSIONS

The conclusions drawn from the data in Table 2 are as follows:

- The presence of a tree root made the linear system of equations harder to solve with some solver and preconditioner options failing when a tree was present. In particular, no preconditioner worked with CR when a tree was present.
- The ASM preconditioner did not work with either CG or CR for any of the datasets.
- The Boomer AMG preconditioner performed significantly better than the other preconditioners.
- The Boomer AMG preconditioner performed equally well with CG, BI, and GM.
- BI and GM took longer to run than CG and CR.

Conjugate Gradient (CG)			
Tree Root	**Preconditioner**	**Iterations**	**Time (sec)**
No	Jacobi	11,045	36.5
Yes	Jacobi	-	-
No	Block Jacobi	3,204	17.2
Yes	Block Jacobi	-	-
No	ASM	-	-
Yes	ASM	-	-
No	Boomer AMG	51	5.8
Yes	Boomer AMG	70	7.4
Conjugate Residual (CR)			
No	Jacobi	10,068	35.1
Yes	Jacobi	-	-
No	Block Jacobi	2,995	16.9
Yes	Block Jacobi	-	-
No	ASM	-	-
Yes	ASM	-	-
No	Boomer AMG	52	5.9
Yes	Boomer AMG	-	-
Bi-CG Stabilized (BI)			
No	Jacobi	11,152	73.6
Yes	Jacobi	15,007	101.0
No	Block Jacobi	3,034	33.3
Yes	Block Jacobi	3,364	38.3
No	ASM	8,779	122.2
Yes	ASM	9,365	130.6
No	Boomer AMG	28	6.2
Yes	Boomer AMG	30	6.6
GMRES (GM)			
No	Jacobi	-	-
Yes	Jacobi	-	-
No	Block Jacobi	33,572	344.0
Yes	Block Jacobi	33,022	322.7
No	ASM	16,830	194.2
Yes	ASM	20,070	230.2
No	Boomer AMG	49	5.9
Yes	Boomer AMG	51	6.1

Table 2: Performance of preconditioners and solvers

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